Short Papers

Cutoff Frequencies of Transmission Lines Consisting of Pair of Cylinders

B. N. Das, S. B. Chakrabarty, and S. Das

Abstract—**The paper presents a method of evaluation of cutoff frequencies of higher-order modes of transmission line of parallel cylinders by transforming them into a parallel plate configuration using cotangent hyperbolic transformation. Application of the method of finite difference to the weighted Helmholtz equation leads to a set of simultaneous equations. The eigenvalues related to cutoff frequencies are determined from the characteristic equation expressed in terms of matrices obtained from the simultaneous equations. Numerical data are presented.**

I. Introduction

In the evaluation of cutoff frequencies of higher-order modes of transmission lines consisting of parallel cylinders, bilinear transformation in terms of mutually inverse points was used [1]. But this was not combined with any powerful numerical method [2] for accurate evaluation of higher-order mode cutoff frequencies. Use of this transformation which transforms parallel cylinders into coaxial configuration has the limitation that, it is difficult to visualise the additional boundary conditions which reveal the characteristics of symmetric and asymmetric higher-order modes resulting from the displacement of the axis of the inner cylinder [3]. Kuttler's transformation [4] which transforms the conductor boundaries to parallel plates is free from this limitation.

In this paper, the weighted Helmholtz equation resulting from Kuttler's transformation is solved by the method of finite difference. The entire region between the transformed parallel plates is divided into rectangular grid for the application of the method of finite difference. The numerical data on cutoff frequencies of eccentric coaxial line evaluated by this method has shown excellently good agreement with those reported in the literature [4]. The same technique is therefore employed for evaluation of cutoff frequencies of transmission lines consisting of parallel cylinders of unequal radii.

II. Analysis

Application of cotangent hyperbolic transformation transforms the structure of Fig. 1 to the configuration shown in Fig. 2. The transformed parallel plates intersect the real axes of the transformed complex plane on its positive and negative sides (Fig. 2).

The expressions for x_1 and x_2, the points of intersection of the parallel plates with the real axes are of the form

$$x_1 = \cosh^{-1} \frac{1 - \left(\frac{R_2}{R_1}\right)^2 + \left(\frac{D}{R_1}\right)^2}{2\frac{D}{R_1}},$$

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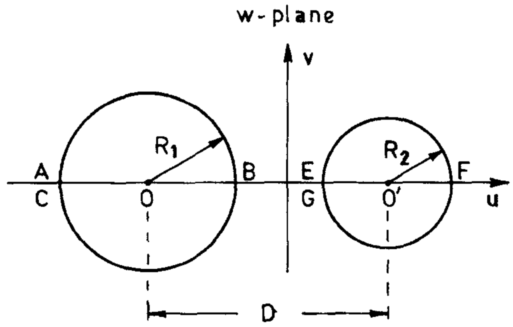

Fig. 1. w-plane representation of parallel wire line.

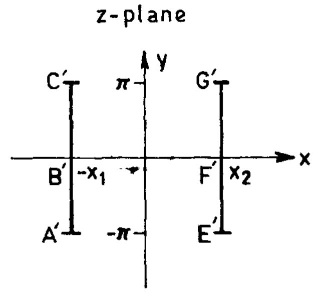

Fig. 2. z-plane representation of the structure of Fig. 1 obtained through conformal transformation.

$$x_2 = \cosh^{-1} \frac{\left(\frac{R_2}{R_1}\right)^2 + \left(\frac{D}{R_1}\right)^2 - 1}{2\frac{R_2 D}{R_1^2}}.$$

The weighted Helmholtz equation resulting from the conformal transformation is obtained as [4], [5]

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + k^2 R_1^2 \left\{ \frac{\sinh(x_1)}{\cosh(x) - \cos(y)} \right\}^2 \psi = 0. \quad (1)$$

Since this equation can not be solved by the method of separation of variables, the dimensionless parameter kR_1 is obtained from the solution of simultaneous equations resulting from the application of the method of finite difference. The entire region of Fig. 2 is divided into rectangular grid as shown in Fig. 3. If M and N are the number of nodes along the x and y directions respectively, the separation between the nodes in the two directions are $h_x = (x_1 + x_2)/M$ and $h'_y = 2\pi/N$.

At the nodes of Fig. 3, the unknown function ψ are represented by $\psi_{(\eta-1)M+\xi}$ where $1 \le \eta \le N$ and $1 \le \xi \le M$. Hence, for $y = -\pi$ and π the potential functions are represented by $\psi_1, \cdots, \psi_M$ and $\psi_{(N-1)M+1}, \cdots, \psi_{NM}$ along the x-axis.

In view of singularity at $x = 0$ and $y = 0$, (1) is multiplied by ($\cosh\, x - \cos\, y$). Following the procedure suggested in the literature [6] the difference equation reduces to the form

$$\begin{aligned}(-\psi_{\eta M+\xi} h_x^2 - \psi_{(\eta-2)M+\xi} h_x^2 - \psi_{(\eta-1)M\xi+1} h_y^2 \\ - \psi_{(\eta-1)M+\xi-1} h_y^2)(\cosh(x_\eta) - \cos(y_\xi)) \\ + [(2h_x^2 + 2h_y^2)(\cosh(x_\xi) - \cos(y_\eta)) \\ - k^2 h_x^2 h_y^2 R_1^2 \sinh^2 x_1]\psi_{(\eta-1)M+\xi} = 0 \quad (2)\end{aligned}$$

where $x_\xi = -x_1 + (\xi - 1)h_x$ and $y_\eta = -\pi + (\eta - 1)h_y$

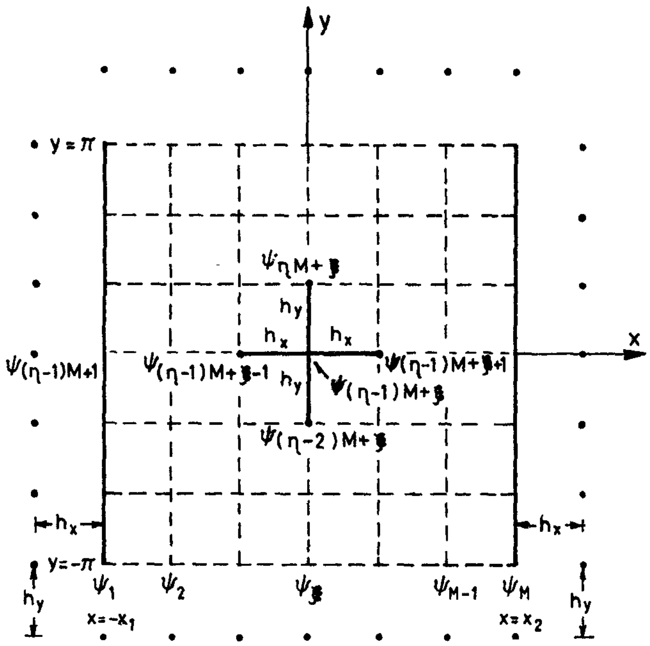

Fig. 3. Geometry of the grid nodes in the transformed structure of Fig. 2.

TABLE I
TM FREQUENCIES k_j FOR ECCENTRIC ANNULAR GUIDE

j	Symmetry	$\frac{R_2}{R_1} = 0.25, \frac{D}{R_1} = 0.5$		
		Present method	Method of ref.[4] Lower bound	Method of ref.[4] Upper bound
1	s	2.9811	2.887	2.996
2	a	3.9945	3.858	4.043
3	s	4.7872	4.088	4.827
4	a	5.5064	4.58	5.575
5	s	5.7897		5.877
6	a	6.2513		6.323
7	s	6.9381		6.992
8	a	7.2544		7.208
9	s	7.5896		7.735
10	a	7.9918		8.166

Equation (2) gives a set of simultaneous equations as η and ξ assumes values over the range $1 \leq \eta \leq N$ and $1 \leq \xi \leq M$ as stated above. The values of M and N are so chosen that x_ξ and x_η never assumes values equal to zero simultaneously. This set of simultaneous equations leads to a matrix equation of the form

$$([A] - k^2 R_1^2 h_x^2 h_y^2 [B])[\psi] = [0] \tag{3}$$

where $[A]$ is a square matrix whose diagonal elements are $2(h_x^2 + h_y^2)$ and nondiagonal elements are $0's$, $-h_x^2$ and $-h_y^2$ as dictated by (2) and the boundary conditions. $[B]$ is a diagonal matrix whose diagonal elements are $\sinh^2 x_1$. $[\psi]$ is a column matrix and [0] is a null column matrix. Representing the eigenvalue of (3) as $\lambda = k^2 R_1^2 h_x^2 h_y^2$ the matrix equation assumes the form

$$([A] - \lambda[B])[\psi] = [0]. \tag{4}$$

TABLE II
TE FREQUENCIES k_j FOR ECCENTRIC ANNULAR GUIDE

j	Symmetry	$\frac{R_2}{R_1} = 0.15875, \frac{D}{R_1} = 0.379$		
		Present method	Method of ref.[4] Lower bound	Method of ref.[4] Upper bound
2	a	1.7404	1.7330	1.7584
3	s	1.7696	1.7603	1.7948
4	a	2.9201	2.873	2.989
5	s	2.9292	2.871	3.004
6	a	3.7386	3.432	3.775
7	s	3.8014	3.78	4.17
8	a	4.1993	3.76	4.21

TABLE III
TM FREQUENCIES k_j FOR PARALLEL LINE, $R_2/R_1 = 0.7$

j	Symmetry	$\frac{D}{R_1} = 2.5$	$\frac{D}{R_1} = 4.0$	$\frac{D}{R_1} = 6.0$	$\frac{D}{R_1} = 8.0$	$\frac{D}{R_1} = 10.0$
1	s	0.0408	0.0203	0.0109	0.0089	0.0054
2	a	0.0755	0.0459	0.0352	0.0296	0.0123
3	s	0.1583	0.0783	0.0455	0.0330	0.0302
4	a	0.2273	0.0861	0.0672	0.0528	0.0401
5	s	0.2879	0.1327	0.0811	0.0781	0.0633
6	a	0.3994	0.1747	0.1145	0.0817	0.0737
7	s	0.4346	0.1832	0.1206	0.0941	0.0831
8	a	0.4741	0.2273	0.1393	0.1264	0.1121
9	s	0.6064	0.2335	0.1626	0.1295	0.1292
10	a	0.6771	0.2842	0.1949	0.1358	0.1223

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The eigenvalues λ are found from the characteristic equation

$$\det([A] - \lambda[B]) = 0. \tag{5}$$

TABLE IV
TE FREQUENCIES k_j FOR PARALLEL LINE, $R_2/R_1 = 0.7$

j	Symmetry	$\frac{D}{R_1} = 2.5$	$\frac{D}{R_1} = 4.0$	$\frac{D}{R_1} = 6.0$	$\frac{D}{R_1} = 8.0$	$\frac{D}{R_1} = 10.0$
2	a	0.0833	0.0495	0.0348	0.0222	0.0215
3	s	0.2121	0.1025	0.0738	0.0459	0.0343
4	a	0.2722	0.1623	0.0645	0.0681	0.0492
5	s	0.3612	0.1682	0.1102	0.0753	0.0711
6	a	0.4321	0.2593	0.1362	0.1191	0.0923
7	s	0.4853	0.2685	0.1483	0.1202	0.1017
8	a	0.5242	0.3244	0.1948	0.1301	0.1239
9	s	0.7352	0.3450	0.2111	0.1584	0.1321
10	a	0.8112	0.4594	0.2242	0.2298	0.1933

The characteristic feature of the higher-order modes in parallel wire line is that they split into symmetric and asymmetric modes [3].

The boundary conditions satisfied by the different modes are [7]

$$\psi = 0 \text{ (Dirichlet condition)}, \quad \frac{\partial \psi}{\partial n} = 0 \text{ (Neumann condition)}$$

The symmetric and asymmetric modes are obtained from either of the above boundary conditions [4] along $y = \pm\pi$.

Satisfying the boundary conditions and applying the finite difference representation of the weighted Helmholtz equation at each node of Fig. 3, a set of simultaneous equations are obtained. From the above set of simultaneous equations, the matrix equation of the form $([A] - \xi[B])[\psi] = [0]$ is obtained. Use of this matrix equation and (5) leads to the desired eigenvalues.

III. NUMERICAL RESULTS AND DISCUSSION

For checking the validity of the method, the numerical data on cutoff frequencies are first evaluated for the structure in which the smaller cylinder is completely enclosed by the larger one. The points of intersection of the transformed parallel lines x_1' and x_2' are on the positive side of the real axis of Fig. 2 and their expressions available in the literature [8] are used for the computation. The difference equation in this case is of the same form as (2) with the modification that x_1 is replaced by x_1' and h_x is replaced by $h_x' = (x_2 - x_1)/M$ and $h_y' = h_y = 2\pi/N$. The numerical data on cutoff frequencies of higher-order TE and TM modes of eccentric coaxial line is presented in Tables I and II for $R_2/R_1 = 0.25, 0.158\,75, D/R_1 = 0.5, 0.379$. Since, the data computed by this method lie between the upper and lower bounds of those data evaluated by Kuttler [4] using the method of intermediate problems, the validity of the analysis is established. The accuracy of Kuttler's data has also been verified by Zhang *et al.* [9]. The agreement of the numerical data of Tables I and II gave confidence in the use of the present technique for evaluating cutoff frequencies of structure of Fig. 1.

The numerical data on cutoff frequencies of higher-order modes of the structure of Fig. 1 presented in Tables III and IV are evaluated using (3)–(5) and the appropriate boundary conditions for symmetric and asymmetric modes for $R_2/R_1 = 0.7$ and D/R_1 varying from 2.5 to 10. The accuracy of the results in Tables III and IV is inferred from use of the same CVLRG routine of IMSL available in cyber main frame for the data of Tables I and II.

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Schottky Diodes for Analogue Phase Shifters in GaAs MMIC's

P. R. Shepherd and M. J. Cryan

***Abstract*— A simple Schottky diode structure, which is easily implemented in a foundry gallium arsenide (GaAs) process, is described. This structure occupies very much less area than the usual technique of realising Schottky diodes, using standard FET structures. Two variations of the diode have been characterized and modeled using a standard equivalent circuit. This has been used to design a simple analogue phase shifter based on a loaded-line configuration. The phase shifter was manufactured using a standard foundry process and has shown excellent results in terms of phase shift linearity with tuning voltage, combined with low insertion loss, over the range 2-8 GHz.**

I. INTRODUCTION

Electronically controllable phase shifters have a number of uses, most particularly in the realization of beam-forming circuits for phased array antennas [1]. In recent years, with the growth of GaAs monolithic microwave integrated circuits (MMIC's), techniques have been developed for realising beam-forming circuits in MMIC form to take advantage of their low power/size/weight, particularly for

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